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Fluctuating heat transfer from hot wires in low Reynolds number flow

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An estimate is obtained of the heat transfer from a constant-temperature hot wire in a two-dimensional low Reynolds number flow ($R \ll 1$). The flow has a small sinusoidally fluctuating velocity superimposed on the mean velocity. For the fluctuating components it is shown that at low frequencies the heat transfer is in phase with the velocity, the magnitude of the heat transfer being given by the result for steady heat transfer, but at high frequencies the heat transfer lags behind the velocity, the relative magnitude of the heat transfer decreasing as $(\text{frequency})^{-1}$.

1. Introduction

Calibration of hot-wire anemometers by measuring the heat transfer from the hot wire for various fluid flow velocities is usually performed in uniform turbulence-free flow even when the anemometer is to be used subsequently to sense velocity fluctuations in turbulent flow. Estimates of the relative magnitudes of terms in the equation describing conservation of energy in the fluid show that this form of calibration is acceptable provided that the frequencies of the velocity fluctuations being measured satisfy the restriction $S \ll \sigma R$, where $S = 2\omega a/U_0$, σ and R are the Strouhal, Prandtl and Reynolds numbers, respectively, $2a$ is the diameter of the wire, ω is the frequency of the fluctuation and U_0 is the mean velocity of the flow. The high frequency case, when the restriction is not satisfied, is discussed below.

The form of the restriction suggests that time-dependent effects are only important at very low flow velocities. For very fine wires this implies that the Reynolds number is typically, although not necessarily, less than one. When $R > 1$, however, and since $\sigma \simeq 1$ for standard air, time-dependent terms can be important only if $S > 1$, that is, only if the flow fluctuates over length scales comparable to or smaller than the wire diameter, a situation of little interest in hot-wire anemometry. Only the case $R \ll 1$ is discussed.

Buoyancy effects, which may be important at low Reynolds number, are not included. Thus the analysis given below should be valid in the range $G^{\frac{1}{3}} \ll R \ll 1$, where G is the Grashof number (Mahony 1956; Collis & Williams 1959). In addition, since the fluid is treated as a continuum, the Knudsen number, the ratio of the molecular mean free path to the wire diameter, must be negligibly small (Levey 1959).

2. Equations of motion

Two-dimensional flow around an infinitely long wire of circular cross-section with its axis normal to the flow direction is considered. In this simplified analysis only the equation of energy conservation in the fluid need be used. Cole & Roshko (1954) and Levey (1959) have shown how an Oseen type of argument can be used to simplify the energy equation for $R \ll 1$. The resulting approximate form is

$$k\nabla^2 T - \rho_\infty c_p \left(\frac{\partial T}{\partial t} + U_j \frac{\partial T}{\partial x_j}\right) = 0, \tag{1}$$

where the symbols each have their usual meaning.

The prescribed constant velocity appropriate to the present analysis is $U_j \equiv (U_0 + U_1 e^{-i\omega t}, U_2 e^{-i\omega t})$ with $U_1, U_2 \ll U_0$. The resulting temperature field is written as $T = T_0 + T_1 e^{-i\omega t}$ under the assumption $T_1 \ll T_0$. In hot-wire anemometry the heat transfer associated with T_1 is used as a measure of U_1, the fluctuating velocity component in the mean flow direction. Linearized equations for the steady and fluctuating components of the temperature follow directly:

$$\nabla^2 T_0 - \alpha U_0 \, \partial T_0 / \partial x_1 = 0, \tag{2}$$

$$\nabla^2 T_1 - \alpha \left(\frac{\partial T_1}{\partial t} + U_0 \frac{\partial T_1}{\partial x}\right) = \alpha (U_1 \cos\theta + U_2 \sin\theta) \frac{\partial T_0}{\partial r}, \tag{3}$$

where $\alpha = \rho_\infty c_p / k$, and (r, θ) are cylindrical co-ordinates with $x_1 = r\cos\theta$. The boundary conditions are $T_0 = T_w$ and $T_1 = 0$ at $r = a$, and $T_0 \to T_\infty$ and $T_1 \to 0$ as $r \to \infty$.

3. Heat transfer from the hot wire

A solution of (2) has been given by Cole & Roshko (1954) and Levey (1959). Their expression for the Nusselt number for steady heat transfer from the wire is

$$\begin{aligned} Nu_0 &= -\frac{1}{\pi (T_w - T_\infty)} \int_0^{2\pi} \frac{\partial T_0}{\partial r}\bigg|_{r=a} a \, d\theta \\ &= 2[\ln(2/\lambda a) - \Gamma]^{-1}, \end{aligned} \tag{4}$$

where $\lambda = \frac{1}{2}\alpha U_0$, Γ is Euler's constant and where higher-order terms in the small parameter λa have been neglected. This result was obtained also by King (1914). It has been shown to be in excellent agreement with experimental results when $R \ll 1$ (Collis & Williams 1959).

A solution of (3) can be obtained in a similar way by substituting

$$T_1 = \tau(r, \theta) \exp(\lambda r \cos\theta)$$

and solving the resulting equation for τ by using, for example, a Green's function involving a series of modified Bessel functions. The term of lowest order in λa again can be extracted. The corresponding Nusselt number associated with the fluctuating component of heat transfer is

$$Nu_1 = \lambda \alpha U_1 \ln\left(\frac{\lambda}{\beta}\right) \left[(\lambda^2 - \beta^2)\left(\ln\left(\frac{2}{\lambda a}\right) - \Gamma\right)\left(\ln\left(\frac{2}{\beta a}\right) - \Gamma\right)\right]^{-1}. \tag{5}$$

The total heat transfer from the wire is described by $Nu_0 + Nu_1 e^{-i\omega t}$. To this order the transverse fluctuating velocity component U_2 does not contribute to the heat transfer.

Equation (5) can be rewritten in the form

$$Nu_1 = -i \frac{U_1}{U_0} \frac{1}{Z} \frac{Nu_0^2 \ln(1+iZ)}{4 - Nu_0 \ln(1+iZ)} \tag{6}$$

$$\to \frac{U_1}{U_0} \frac{Nu_0^2}{4} \quad \text{as} \quad Z \to 0 \tag{7}$$

$$\to \frac{U_1}{U_0} \frac{Nu_0}{Z} e^{\frac{1}{2}i\pi} \quad \text{as} \quad Z \to \infty, \tag{8}$$

where $Z = 4S/(\sigma R) = 4\omega k/(U_0^2 \rho_\infty c_p)$. The principal values of the logarithms are implied. Nu_0 is defined by (4).

The low frequency result (7) can be obtained directly from the result (4) for steady heat transfer by substituting a Reynolds number $R(1 + U_1 e^{-i\omega t}/U_0)$ and retaining in the fluctuating component only terms of first order in U_1/U_0. Thus at low frequencies, as expected, the fluctuating heat transfer is in phase with the fluctuating velocity and the steady-state heat-transfer calibration of the hot wire indeed can be used to determine the magnitude of the fluctuating velocity.

At high frequencies, on the other hand, the heat transfer lags behind the fluctuating velocity by $\frac{1}{2}\pi$ and the relative amplitude decreases as $(\text{frequency})^{-1}$. In this range, the time-derivative term in (3) is important. The fluctuations in velocity occur so rapidly that the temperature gradients generated close to the wire are relatively small and unimportant as a means of generating heat flow; thus there is only a small net transfer of heat away from the wire. Numerical estimates of (6) indicate that the minimum value of S at which time derivatives become important is $S \sim \frac{1}{10}\sigma R$.

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